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Inter-edge-mode scattering in a high-mobility strained silicon two-dimensional electron system

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Abstract. The magnetoresistances of two-dimensional electron systems in strained silicon on a relaxed silicon–germanium buffer have been measured at low temperatures (50 mK). Samples, with Hall mobilities up to 3.61×10^5 cm^2 V^{-1} s^{-1}, have shown a marked asymmetry between adjacent Shubnikov–de Haas peaks and a prominent overshoot on the low-field side of the odd-filling-factor quantum Hall plateaux. This effect persisted to unusually small magnetic fields. It is argued that both of these phenomena can be explained by a strong back-scattering of multiple edge modes which is suppressed at integer filling factors.

1. Introduction

The edge-state model of the quantum Hall effect behaviour of two-dimensional electron systems (2DESs) in strong magnetic fields has proven to be very successful for explaining a range of phenomena. It considers the probability of transmission between ohmic contacts of one-dimensional channels formed as a result of magnetic quantization, as described by the Landauer–Büttiker formalism [1]. The picture must be extended to account for the transition between integer Landau-level filling factors, when the Fermi energy (E_F) passes through the centre of an energy band, allowing charge transport between sample edges through bulk states in this band. The decoupling of the bulk mode from the edge modes leads to non-local transport behaviour [2], so the standard definition of resistivity no longer describes an intensive (geometry-independent) quantity. The experiments are nonetheless well described by a model which treats the edge and bulk modes separately [2, 3].

Most investigations of these phenomena have employed 2DESs in inversion layers of silicon metal–oxide–semiconductor structures, or in GaAs/AlGaAs heterostructures in which much higher mobilities are attainable. It has recently become possible to achieve comparable mobilities in silicon by using heterostructures of Si and SiGe [4]. To obtain the required conduction-band offset in these materials, it is necessary to use a strained Si channel on a relaxed SiGe buffer, which can be achieved by growing a thick, graded-composition layer on a silicon substrate to allow the strain arising from the lattice mismatch to relax by formation

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of misfit dislocations. Despite the inevitable dislocations present in these structures, low-temperature mobilities as high as 3.9×10^5 cm^2 V^{-1} s^{-1} (at a carrier density of 4.8×10^{11} cm^{-2}) have been obtained in as-grown samples [5].

In this paper, results are presented for samples of this type in which evidence is found for a strong coupling between edge modes and bulk modes at non-integer filling factors, leading to partial back-scattering of modes in addition to the uppermost one.

2. Experiment and results

The methodology behind the growth of the material discussed in this paper has been described previously [6]. Magnetotransport measurements performed in a dilution refrigerator are presented in figures 1 and 2 for a sample of wafer 7B22G15. The sample has a Hall mobility of 3.61×10^5 cm^{2}V^{-1}s^{-1} with a carrier density of 3.24×10^{11} cm^{-2} at 50 mK, which is one of the highest mobilities yet reported in these materials. The plots show a number of interesting features, many of which have been noted before in similar material [7–11]. The Shubnikov–de Haas oscillations display splitting due to lifting of both spin and then valley degeneracies, while at higher fields, fractional quantum Hall effect states are observed at filling factors of 4/3 and approximately 8/5.

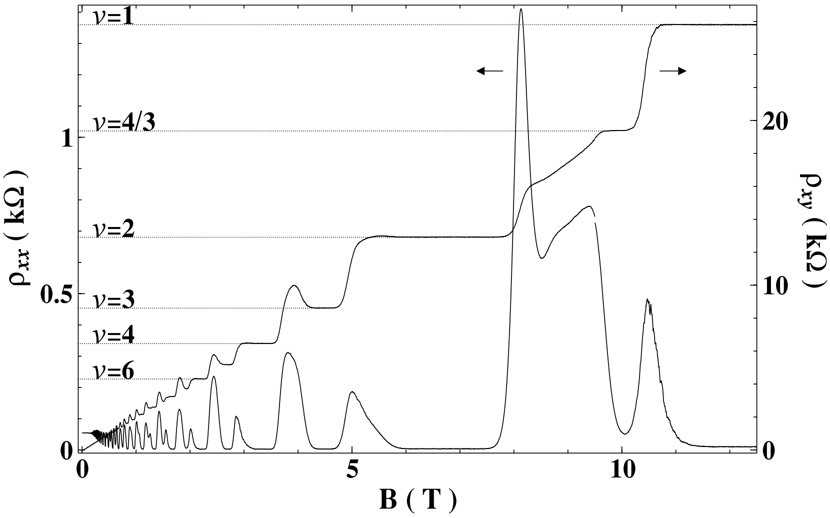

Figure 1. The longitudinal and Hall magnetoresistivity at 50 mK for a Hall bar sample of wafer 7B22G15.

There are also some features which are less universally observed. Inspection of the adjacent pairs of Shubnikov–de Haas peaks corresponding to valley-split levels (such as those at Landau-level filling factors $\nu = 4.5$ and $\nu = 5.5$) reveals a strong asymmetry, with the higher-field peak, corresponding to the lower-energy level (at $\nu = 4.5$ in this example), being considerably weaker. Figure 2 shows that the weaker peaks become equivalent in strength as the temperature is raised. Simultaneously, one observes a strong overshoot feature in the Hall resistance at the low-field end of the odd-numbered plateaux. This feature is particularly strong at low fields, even when the valley splitting is no longer resolved and the odd plateaux are not observed. These two phenomena have also been observed, though more weakly, in a Schottky-gated sample from the same wafer [12], and in a sample of another similar wafer

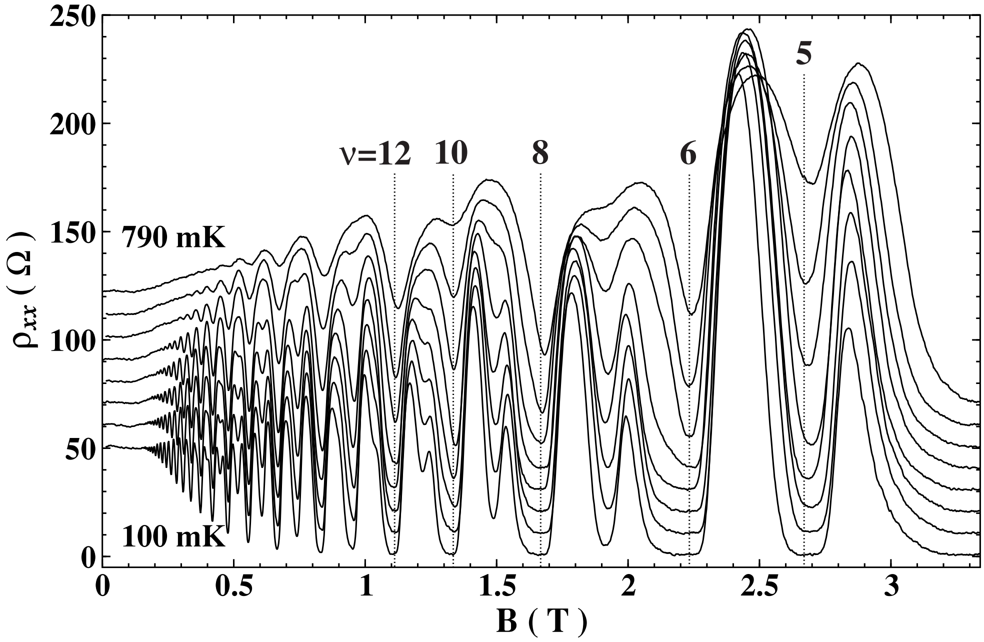

Figure 2. The low-field longitudinal magnetoresistivity for wafer 7B22G15 at the following temperatures: 100 mK, 160 mK, 200 mK, 300 mK, 350 mK, 450 mK, 590 mK and 790 mK. The curves are offset in steps of 10 Ω for clarity.

(6B34G15) [13] which has a mobility of 2.81×10^5 cm^2 V^{-1} s^{-1} at a carrier density of 4.35×10^{11} cm^{-2}. In both cases, the Hall overshoots again become strong at low fields.

2.1. Shubnikov–de Haas asymmetry

Asymmetries between pairs of Shubnikov–de Haas peaks have been observed previously in both silicon inversion layers [14] and GaAs/AlGaAs layers [15]. These early results were ascribed to a low density of attractive impurities which cause a low-energy tail in the density of states of each energy level. This tail is expected to overlap in energy with a nearby level, reducing the conductivity of the bulk states in that level. Hence, back-scattering via those bulk states is suppressed, and a reduced resistivity peak appears when the Fermi energy lies in this energy band. Evidence has been put forward which confirms the validity of this explanation for some samples [15, 16].

This model also predicts that the asymmetric density of states should give an asymmetry in the shape of individual peaks [17] which is weakly observed here: it is most noticeable for the peaks close to 3 T and 5 T in figure 1.

Various measurements have suggested that this model cannot completely explain all the observations of asymmetries; notably, it fails to account for the observation that the asymmetry is stronger in narrow or long samples [18, 19]. An alternative mechanism has therefore been suggested, which involves scattering between adjacent edge channels [19].

Although the orthogonality of the edge modes should prevent transitions between them, it has been shown in experiments which selectively probe certain edge states that all the modes aside from the one closest to the Fermi energy (the nth) rapidly equilibrate when current is injected unevenly among them [20, 21]. The lack of equilibration with the nth mode results from the fact that as it approaches E_F, this mode moves in towards the bulk of the sample, reducing the spatial overlap with the true edge modes [22].

In this picture, the ρ_{xx}-peak arises purely from back-scattering of the nth mode. When the $(n-1)$th level is close in energy to the nth, however, there may be back-scattering from this mode too, due to scattering between these two modes. This will lead to a larger resistance peak for the higher-energy level of a closely spaced pair, because of the proximity of the nth and the $(n-1)$th levels in this case [22], as observed in the data (figure 1). The energy spacings for even-integer-filling-factor gaps are larger, so enhancement of the lower valley peak should only occur when the levels are broadened by raising the temperature. This effect is observed in the temperature dependence shown in figure 2.

The main phenomenological difference between these models is that the impurity model predicts that the low-energy peak should be suppressed, while the edge-state coupling model predicts that the high-energy peak is enhanced. This distinction may be tested by considering the absolute peak height. If only the topmost edge mode (or the m topmost modes) are partially back-scattered, it is possible to place an upper limit on the resistance which can be observed. If the measured resistance exceeds this value, one can say that more than one mode (or m modes) are back-scattering.

This limit is determined by considering the case when m edge modes are completely reflected in the region between the voltage probes, with no reflection occurring elsewhere in the sample. Since this is not a realistic situation for the samples being measured, one would expect the actual resistance to fall somewhat short of the upper limit:

$$R_{\max} = \frac{h}{e^2}\left(\frac{1}{n-m} - \frac{1}{n}\right). \tag{1}$$

Note that this is an extensive quantity which should, therefore, be compared with the measured *resistance* rather than the resistivity.

This formula, with $m = 1$ and $m = 2$, is compared with the experimental peak maxima for wafers 7B22G15 and 6B34G15 in figure 3. Data are only shown for the field range in which zeros between valley-split ρ_{xx}-peaks are observed.

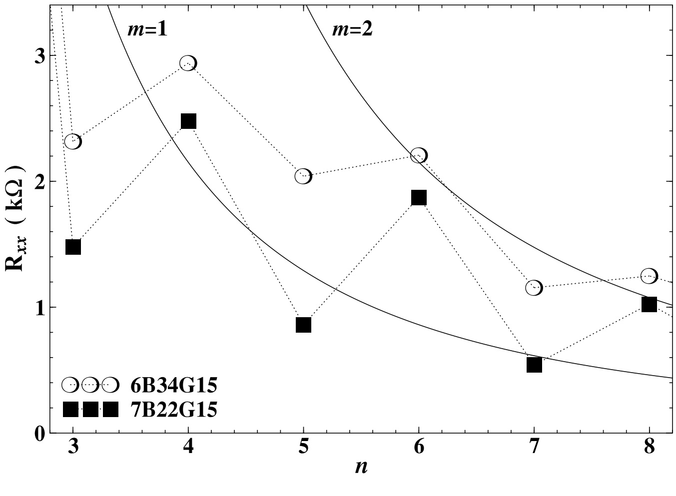

Figure 3. R_{xx}-maxima for wafers 7B22G15 and 6B34G15 at 50 mK as a function of the number of occupied and partially occupied edge channels, n. Also shown are theoretical maximum resistances for the cases of $m = 1$ or 2 completely back-scattered edge channels (solid lines).

The odd-numbered peaks from wafer 7B22G15 are below the $m = 1$ line, indicating that back-scattering from the inner channel is not complete, as should be expected, since the sample is not infinitely narrow or long. The even peaks on the other hand exceed this value, clearly indicating that there is back-scattering from other edge modes. The two sets of peaks approach the theoretical values for $m = 1$ and $m = 2$ at large n, that is, as the energy spacing of the levels becomes smaller. This may indicate that there is near-complete back-scattering of the valley-split pairs at low fields, but that other spin or orbital levels are decoupled within this field range, giving the observed asymmetry. The increasing peak resistances might, however, simply arise from contributions to the total back-scattering from additional edge modes, as their separation is reduced.

For wafer 6B34G15, both sets of peaks show higher resistances, suggesting that there is certainly coupling with other levels in this case, even though there are still clear zeros between the ρ_{xx}-peaks. This enhanced scattering may be related to the lower mobility, since inter-edge-state transitions require the presence of impurities (or phonons) [23]. This is consistent with the fact that the only other reported Si/SiGe 2DES sample showing a strong asymmetry also had a large mobility (3.2×10^5 cm^2 V^{-1} s^{-1}) [11].

This is strong evidence that inter-edge-state scattering occurs in these samples even when there are clear zeros in ρ_{xx}. This does not rule out the presence of some effect due to impurities, which may be the cause of the slight asymmetry of some individual peaks.

2.2. Hall overshoot

Turning now to the Hall overshoots, which are shown in greater detail and with a temperature dependence for wafer 7B22G15 in figure 4, there are a number of possible explanations to consider.

The first possibility to rule out is that this arises simply due to admixture of ρ_{xx} as a result of a geometrical asymmetry in the sample. If this were the case, one should expect the admixture to be reversed when the magnetic field is reversed. Traces for the two field directions are shown in the lower inset to figure 4. It is evident that over much of the field range examined, the size of the overshoot varies with field direction, indicating some geometrical effect, but the direction of the overshoot is not reversed. Also, at low fields, there is little dependence on the field direction, so the overshoots cannot be purely due to admixture.

It has also been shown [24] that overshoots may occur in the Hall resistivity as a result of the oscillation described by semi-classical theory. This can be related to the oscillation in the longitudinal resistivity by the expression

$$\Delta\rho_{xy} = -\frac{\Delta\rho_{xx}}{2\mu B}. \tag{2}$$

Clearly, the oscillations in ρ_{xy} will be strongest in samples with low mobility and indeed, it is found that the calculated $\Delta\rho_{xy}$ is far too small in the sample of 7B22G15 to account for the observed effect.

A third possible explanation is related to the inter-edge-state scattering used to explain the Shubnikov–de Haas asymmetry. In this picture, at an even-integer filling factor, n, if the field is increased, there is back-scattering from both levels of a valley-split pair. This means that the Hall resistance should increase from h/e^2n towards $h/e^2(n-2)$. The fact that ρ_{xy} returns to $h/e^2(n-1)$ at integer filling factor indicates that the Fermi energy must lie in an energy gap at this point.

This description relies on the existence of a variable valley splitting, since the modes must overlap at the edge, but be well separated in the bulk. This could just be a spatial variation arising from the confining potential at the edge, as pointed out by Komiyama *et al* [3], but

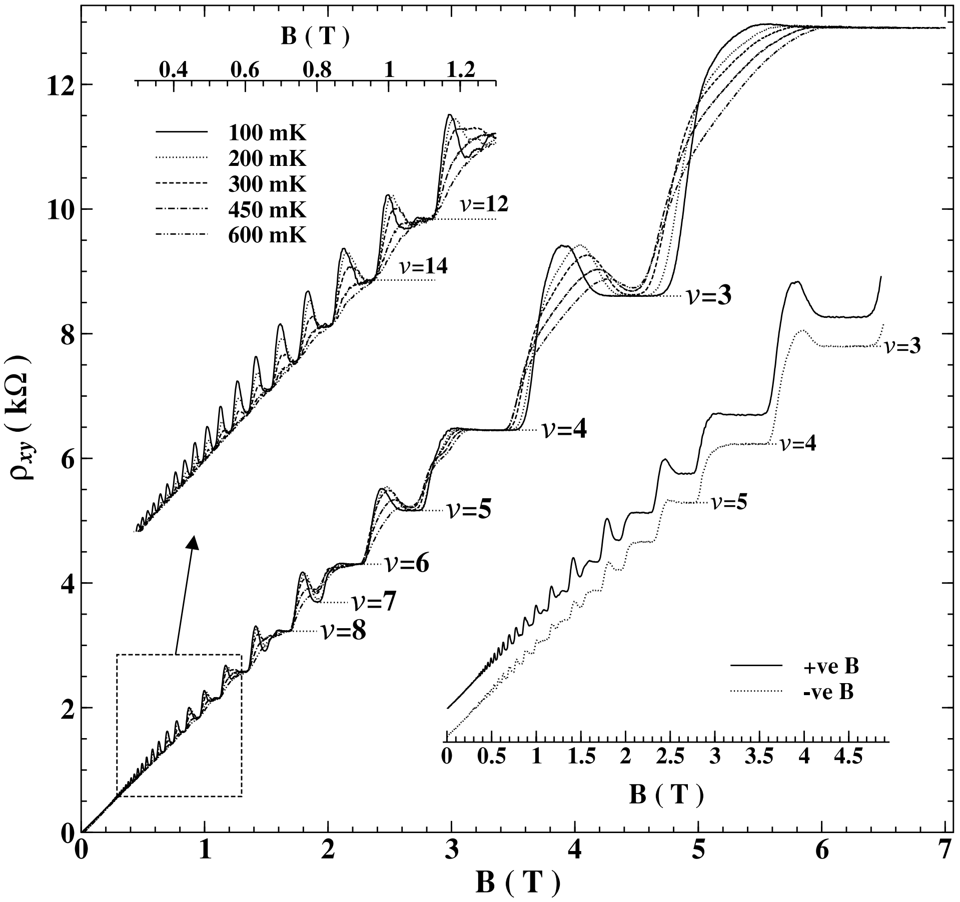

Figure 4. The temperature dependence of ρ_{xy} for wafer 7B22G15, showing the evolution of the overshoots. The top left inset is an enlargement of the low-field data, while the bottom right inset demonstrates the effect of reversing the field direction (curves offset for clarity).

there may also be a field dependence, as suggested by Richter *et al* [25]. The authors of reference [25] explained similar-looking overshoots in a GaAs/AlGaAs sample as described above, but with an exchange enhancement of the spin splitting giving rise to the energy gap at odd-integer filling factor. In these Si/SiGe samples, a similar exchange enhancement of the small valley gap may cause the overshoot to return to the plateau resistance.

3. Discussion

The results for wafer 7B22G15 have much in common with those presented in references [3] and [25], since an asymmetry in the Shubnikov–de Haas peaks was also observed in those cases. A distinction between the results for wafer 7B22G15 and the cited results is that the latter only show overshoots for a very limited range of $g\mu_B B/k_B T \approx 1$. This is understandable, since the energy gap is too large for inter-edge-state scattering to occur at larger fields, while at low fields, the gap is too small for back-scattering to be suppressed at integer filling factors.

This is consistent with the absence in the present samples of an overshoot at the $\nu = 1$ plateau, which would, in any case, be suppressed by additional energy gaps associated with the fractional quantum Hall states. On the other hand, the peaks at low fields become increasingly prominent. This suggests that the energy gap is not related directly to the magnetic field and that, at least for the valley splitting, the exchange enhancement of the gap plays an important role in these materials.

The fact that the overshoot persists when the valley splitting is no longer resolved in ρ_{xx} and the fact that the low-field overshoots extend beyond the $\nu = n - 2$ plateaux suggests that the effect is not confined to scattering to an adjacent valley-split level. From the overshoot peak heights, there appear to be up to at least four non-degenerate edge modes back-scattering simultaneously at low fields. This indicates a large amount of mode overlap at the edges and the presence of spin and/or orbital transitions. The suppression of this back-scattering at integer filling factor can still be explained provided an energy gap exists in the bulk of the sample. This will allow ρ_{xy} to return to the quantized value and give the observed oscillation [3].

A possible argument against this explanation is the T-dependence which shows the effect becoming stronger as the temperature is lowered, in contrast to the results of Richter *et al* [25]. If the back-scattering from outer edge modes is limited by scattering to the bulk mode, which should increase as the temperature is raised, one would expect the overshoots to become stronger at higher temperature. Therefore, if the results are to be explained within the edge-state coupling model, one must infer that the back-scattering of the outer modes is limited instead by the intra-mode back-scattering across the device. In this case the similarity of the ρ_{xx}- to the ρ_{xy}-enhancements is to be expected, since they are both governed by the conductance of the bulk states.

This inference and the strength of the overshoots both require that the scattering between the bulk mode and the edge modes is very strong in these samples. This has been shown to be the case for inter-valley scattering from the Shubnikov–de Haas asymmetry discussed in the previous section. The large Hall overshoots at low field indicate that this is also the case for spin and/or orbital transitions.

It is unclear why these should be stronger than similar transitions in GaAs/AlGaAs 2DESs. There have been few reports of transport measurements of Si/SiGe 2DES materials below 100 mK in the literature, so it is difficult to know how universal the phenomenon is in these structures. Only Weitz *et al* [11] report any overshoot: a small feature at $\nu = 3$ which was found to be enhanced by a tilted field. This was ascribed to crossing of energy levels, which would clearly enhance scattering between the coincident modes.

In conclusion, strong asymmetries have been observed in both resistivity components in high-mobility strained silicon 2DES samples. These effects extend over a much wider field range than has been previously reported for any material and have been explained as resulting from a strong inter-edge-state scattering leading to back-scattering of multiple edge modes.

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